Orissa Journal of Physics
ISSN 0974-8202

Vol. 24, No.1
February 2017
pp. 127-132

Deformed Shell Model study of LSP Detection Rates with ^{73}Ge as the Detector

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Received: 2.12.2016 ; Revised : 3.12.2016 ; Accepted :15.1.2017

Abstract. The detection rates for the lightest super symmetric particle (a dark matter candidate) are calculated with ^{73}Ge as the detector. The calculations are performed within the deformed shell model (DSM) based on Hartree-Fock states. First the energy levels and ground state magnetic moment for ^{73}Ge are calculated and compared with experiment. The agreement is quite satisfactory. Then the nuclear wave functions are used to calculate the detection rate as a function of detector threshold energy for a given set of SUSY parameters. The results are compared with other theoretical calculations.

Keywords: Dark mater detection, deformed shell model,

PACS Number: 95.35.+d, 21.60.Jz, 27.50.+e

1. Introduction

There has been overwhelming evidences for the existence of dark matter in the universe [1, 2]. The evidences mainly come from the rotational curves of spiral galaxies, gravitational lensing in clusters of galaxies, anisotropy in the cosmic microwave background radiation etc. Up to now, the nature of this matter remains a mystery. In recent years, there have been considerable theoretical and experimental efforts to detect the cold dark matter (CDM) which is thought to be the dominant component of the dark matter [3]. Super symmetric theories of physics beyond the standard model provide the most promising candidates for dark matter. In the simple picture, the dark matter in the galactic halo is assumed to be Weakly Interacting Massive Particles (WIMP). The most appealing WIMP candidate for nonbaryonic ^{73} Ge cold dark matter is the lightest super symmetric particle (LSP) which is expected to be a neutral Majorana fermion travelling with nonrelativistic velocities.

Since the LSP (represented by χ) interacts very weakly with matter, its detection is quite difficult. One possibility to detect LSP is through its elastic scattering from nuclei. Inelastic channels are not excited since the energy is too low to excite the nucleus and hence the cross section should be negligible. On the other hand exotic WIMPs can lead to large nucleon spin induced cross sections which in turn can lead to non-negligible probability for inelastic WIMP-nucleus scattering [3]. Here we will consider only the elastic channels. The deformed shell model (DSM), based on Hartree-Fock (HF) deformed intrinsic states with angular momentum projection and band mixing, is established to be a good model to describe the properties of nuclei in the mass range A=60-100. Also, for N=Z odd-odd nuclei, methods for isospin projection within DSM are developed and applied. See [4] for details regarding DSM. The model is found to be quite successful in describing spectroscopic properties, double beta decay half-lives, μ -e conversion in the field of the nucleus and so on. It will be quite interesting to employ DSM to calculate the detection rates for the lightest super symmetric particle (a dark matter candidate) with ^{73}Ge as the detector.

2. Formulation

Defining the dimensionless quantity $u = q^2 b^2 / 2 = M_A b^2 \bar{Q}$ where q represents the momentum transfer to the nuclear target, b is the nuclear harmonic oscillator size parameter, $\bar{Q}$ is the energy transfer to the nucleus and M_A is the nuclear mass, the LSP-nucleus differential cross section in the laboratory frame is given by [5, 6],

$$\frac{d\sigma(u,v)}{du} = \frac{1}{2}\sigma_0 \left(\frac{1}{m_p b}\right)^2 \frac{c^2}{v^2} \frac{d\sigma}{v^2} \frac{d\sigma_{AS}(u,v)}{du} \; ; \qquad (1)$$

$$\frac{d\sigma_{AS}(u,v)}{du} = \left[f_A^0 \Omega_0(0) \right]^2 F_{00}(u) + 2 f_A^0 f_A^1 \Omega_0(0) \Omega_1(0) F_{01}(u)$$
$$+ [f_A^1 \Omega_1(0)]^2 F_{11}(u) + M^2 . \qquad (2)$$

In the above, m_p is the mass of the proton, v is the LSP velocity with respect to the earth and $\sigma_0 = 0.77 \times 10^{-35}\,\text{cm}^2$. If the proton and neutron form factors $F_Z(u)$ and $F_N(u)$ are different, then

$$M^2 = (f_s^0 [Z F_Z(u) + N F_N(u)] + f_S^1 [Z F_Z(u) - N F_N(u)])^2 . \qquad (3)$$

Here, f_A^0 and f_A^1 represent isoscalar and isovector parts of the axial vector current and similarly f_S^0 and f_S^1 represent isoscalar and isovector parts of the scalar current. These nucleonic current parameters depend on the specific SUSY model employed. The spin structure functions $F_{\rho\rho'}(u)$ with $\rho, \rho' = 0,1$ are defined as

$$F_{\rho\rho'}(u) = \sum_{\lambda,\kappa} \frac{\Omega_\rho^{(\lambda,\kappa)}(u)\Omega_{\rho'}^{(\lambda,\kappa)}(u)}{\Omega_\rho(0)\Omega_{\rho'}(0)} ;$$

$$\Omega_\rho^{(\lambda,\kappa)}(u) = \sqrt{\tfrac{4\pi}{2J_i+1}}$$

$$\times \left\langle J_f \left\| \sum_{j=1}^{A} [Y_\lambda(\Omega_j) \otimes \sigma(j)]_\kappa \, j_\lambda(\sqrt{u}\, r_j) \times w_\rho(j) \right\| J_i \right\rangle \quad (4)$$

With $w_0(j) = 1$ and $w_1(j) = \tau(j)$; note that τ = +1 for protons and -1 for neutrons. Here Ω_j is the solid angle for the position vector of the j-th nucleon and j_λ is the spherical Bessel function. The static spin matrix elements are defined as $\Omega_\rho(0) = \Omega_\rho^{(0,1)}(0)$. As has been described in [5], the LSP detection rate is given by the simple expression,

$$R_0 = 8.9 \times 10^7 \times \frac{\sigma_{AS}(v_{esc})}{Am_x[GeV](m_p b)^2}[yr^{-1}kg^{-1}] . \quad (5)$$

Note that $v_{esc} = 625/s$ is the escape velocity of the LSP from the milky way and the LSP mass m_χ is taken to be 110 GeV. The $\sigma_{AS}(v_{esc})$ is obtained using Eq. (2) and the Maxwell velocity distribution (for v). In the integral over u, the lower limit involves the detector threshold energy Q and the upper limit involves v_{esc}.

The nuclear structure part is in the spin structure functions and the form factors. It is here DSM is used. The reduced matrix element appearing in Eq. (4) can be evaluated in DSM. Here we need the sp matrix elements of the operator of the form $t_v^{(l,s)J}$ and these are given by,

$$\left\langle n_i l_i j_i \left\| \hat{t}^{l,s),J} \right\| n_k l_k j_k \right\rangle = \sqrt{(2j_k+1)(2j_i+1)(2J+1)(s+1)(s+2)}$$

$$\begin{Bmatrix} l_i & \frac{1}{2} & j_i \\ l_k & \frac{1}{2} & j_k \\ l & s & J \end{Bmatrix} \left\langle l_i \left\| \sqrt{4\pi} Y^l \right\| l_k \right\rangle \left\langle n_i l_i \left\| j_l(kr) \right\| n_k l_k \right\rangle . \tag{6}$$

In the above equation, {--} is the nine-*j* symbol.

3. Results and discussion

Above formulation is used for LSP detection rates for scattering from ^{73}Ge with DSM for the nuclear structure part. The sp orbits employed are $^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, and $^1g_{9/2}$, with ^{56}Ni core and the sp energies are taken as 0.0, 0.78, 1.08 and 4.90 MeV respectively. The effective interaction used is the modified Kuo interaction [8]. The HF sp spectrum is shown in Fig. 1a. For ^{73}Ge, the experimental energy spectrum has positive and negative parity levels at low energy. Hence, for band mixing in DSM three intrinsic states with positive parity and three with negative parity are considered. The final energy spectrum and its comparison with experiment is shown in Fig. 1b. Since spin contributions play an important role in the calculation of the decay rates, the magnetic moment is decomposed into orbital and spin parts for this nucleus. The DSM value for the magnetic moment (using bare values for the *g*-factors) is -0.811 μ_N and it is close to experimental value -0.879 μ_N [7]. The matrix elements of the proton orbital and spin angular momenta are 0.581 and -0.001 respectively and similarly, for neutron the values are 3.558 and 0.362 respectively.

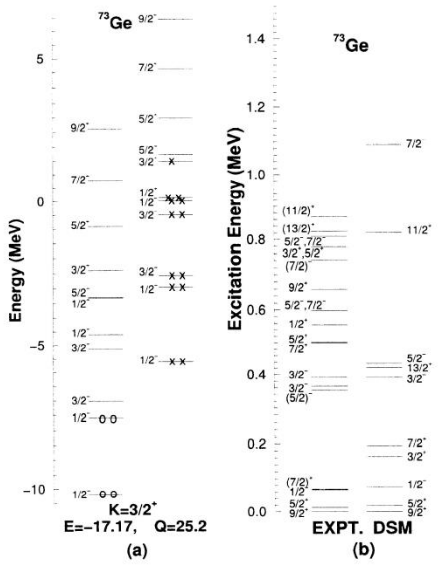

Fig.1: (a) The lowest prolate HF sp spectrum for ^{73}Ge. The HF energy E in MeV and the mass quadrupole moment Q in units of the square of the oscillator length parameter b are also given. Protons are represented by circles and neutrons by crosses. **(b)** Theoretical (DSM) and experimental (EXPT) spectra of ^{73}Ge. Data are taken from [7]

Depending on the SUSY parameters, the detection rate varies widely as described in [5]. The same feature is also found in DSM. The values of the parameters f_A^0, f_A^1, f_S^0 and f_S^1 are taken from [9] and they are $3.55\times10^{-2}, 5.31\times10^{-2}, 8.02\times10^{-4}$ and $-0.15\times f_S^0$ respectively. For ^{73}Ge, DSM gives the values of Ω_0 and Ω_1 to be 0.798 and -0.803. These values are smaller than those quoted in [5], where a quasi-particle-phonon model (QPM) is used, by 20 to 30 percent. The spin structure functions which do not depend on the oscillator length parameter are plotted in Fig. 2. The structure functions for ^{73}Ge are similar to those obtained using QPM in [5]. Ressell et al. [10] calculated $S_{\rho\rho'}$, that are related to the spin structure functions defined above, using shell model. The spin structure functions from DSM are similar to their values. Following these, the detection rate as a function of Q is obtained using Eq. (5) and the results are shown in Fig 2. Results in the figure show that ^{73}Ge is a good detector for detecting dark matter.

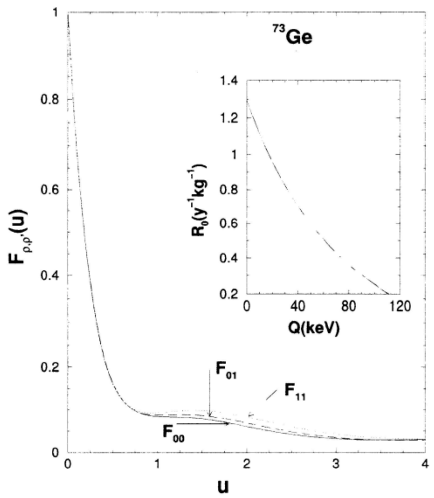

Fig. 2: Spin structure functions for ^{73}Ge as a function of momentum transfer *u*. Shown in the insect figure is LSP detection rate as a function of *Q*, the detector threshold energy.

5. Conclusion

Applications of DSM to study the detection rates for the light super symmetric particle, a dark matter candidate, with ^{73}Ge as the detector is presented in this paper. It is a problem of current interest in nuclear structure. In future, in the topic of dark matter, DSM will be employed to study inelastic (spin dependent) WIMP-nucleus scattering in ^{83}Kr and this is unlike LSP that involves only elastic scattering.

Acknowledgments

The author is thankful to Prof. V.K.B. Kota for collaboration. He is thankful to SERB of Department of Science and Technology (Government of India) for financial support.

References

[1] G. Jungman, M. Kamionkowski and K. Griest, Phys. Rep. 267 (1996) 195-373.

[2] D. Majumdar, Dark Matter: An Introduction (CRC press, Taylor and Francis group, Florida, 2014).

[3] J.D. Vergados, F.T. Avignone III, P. Pirinen, P.C. Srivastava, M. Kortelainen and J. Suhonen, Phys. Rev. D 92 (2015) 015015/113.

[4] V.K.B. Kota and R. Sahu, Structure of Medium Mass Nuclei: Deformed Shell Model and Spin-isospin Interacting Boson Model (CRC Press, Taylor and Francis group, Florida, 2016).

[5] E. Holmlund, M. Kortelainen, T.S. Kosmas, J. Suhonen and J. Toivanen, Phys. Lett. B 584 (2004) 31-39.

[6] P.C. Divari, T.S. Kosmas, J.D. Vergados and L.D. Skouras, Phys. Rev. C 61 (2000) 054612/1-12.

[7] http://www.nndc.bnl.gov/ensdf

[8] D.P. Ahalpara, K.H. Bhatt and R. Sahu, J. Phys. G: Nucl. Phys. 11 (1985) 735-744.

[9] J.D. Vergados, J. Phys. G: Nucl. Part. Phys. 22 (1996) 253-272.

[10] M. Ted Ressell et al., Phys. Rev. D 48 (1993) 5519-5535